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Escherichia coli O157:H7 in food and its health-related risks

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Abstract

E. coli O157:H7 are one of the most important causes of diarrheal episodes. Although the number of *E. coli* O157:H7 infection was decreasing in recent years, the pathogen remains a significant food safety issue. A subgroup of *E. coli*, serotype O157, which was first identified as a human pathogen after the 1982 outbreak of hemorrhagic colitis in Oregon and Michigan, has since been implicated in many cases of food contamination. More than 90% of *E. coli* O157:H7 strains isolated in North America and the UK belong to serotype O157. As the numbers of food–borne human *E. coli* O157 infections have increased, considerable efforts have been made along the food chain to reduce the risk of food contamination.

Keywords: Contamination, food safety, Pathogen, microbiological quality

1 Introduction

The increasing number and severity of food poisoning outbreaks on a global scale have considerably increased public awareness of food safety (Food Standards Agency, 2012). Well publicised cases of Escherichia coli (*E. coli*) serotype O157:H7 infections in particular are of concern due to the potential severity of symptoms (HPA, 2013). Although *E. coli* O157:H7 was only first recognized as a cause of foodborne illness just over 30 years ago (Forsythe, 2010), it has been implicated in sporadic cases and large outbreaks of haemorrhagic colitis and fatal haemolytic uremic syndrome (Karmali et al., 2010). This serotype is regarded as being more transmissible than other E. coli serotypes due to a number of reasons, including its increased tolerance to acid, which allows it to easily survive the acid conditions of the stomach. This bacterium also produces Shiga toxins, which are heat stable, and therefore unaffected by conventional pasteurization methods (Rasooly and Do, 2010). Small doses of fewer than 10 cells may lead to infection (Forsythe, 2010). Collectively, these factors make the control of *E. coli* O157:H7 an important issue in recent times for the food sector.

The main causes for concern and product recalls associated with meat products are *E. coli* O157:H7 (Mor-Mur and Yuste, 2010). In particular, cattle and sheep are major reservoirs for this pathogen (Nastasijevicl et al., 2008; Hutchinson et al., 2005) and contamination of carcasses and food products by animal faeces can lead to transmission of foodborne pathogens to consumers (Oliver et al., 2008). Numerous interventions to be applied at the farm level have been investigated over the past 20 years, but most have proven to be ineffective and/or impractical (Soon et al., 2011).

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2 Clinical aspects of *E. coli* O157:H7

Infection with *E. coli* O157:H7 is asymptomatic in a large proportion of cases, but can also engender a wide range of clinical symptoms ranging from non-bloody diarrhoea to hemorrhagic colitis and other life-threatening complications. Serious health effects arising from infection can involve acute renal failure caused by haemolytic uraemic syndrome (HUS) (Rahal et al., 2012), and neurological problems in the form of thrombotic thrombocytopaenic purpura (TTP) (Duffy et al., 2006; Thomas and Elliott, 2013). Other rare complications include pancreatitis, diabetes mellitus, and pleural and pericardial effusions (Mead & Griffin, 1998). Occasionally, patients infected with *E. coli* O157:H7 suffer damage to their central nervous system as TTP, which typically includes seizures arising from hypertensive encephalopathy. Untreated TTP can have a mortality rate as high as 95%. Symptoms may include thrombocytopenia, fever, renal insufficiency, neurological deficit, microangiopathic haemolytic anaemia, headache, fatigue/malaise, altered mental status, and hemiplegia (Rahal et al., 2012).

3 Prevalence of infections

E. coli O157:H7 has become the most frequently reported cause of bacteraemia in England, Wales and Northern Ireland (HPA, 2007). A report from HPA (2013) (Figure 2.1) suggests almost a 100% increase, from 595 to 1182, in the annual totals of VTEC infections in England & Wales between 2002 and 2011. To date, many parts of the world have witnessed outbreaks of VTEC infections involving serotype O157 (Duffy et al., 2006). Infection rates differ widely between geographical regions. In Europe, Scotland possesses the highest infection rates with approximately 4 cases per 100,000 (Duffy et al., 2006), while in Northern Europe infection rates are very low (e.g. 0.04 per 100,000 in Norway and Finland). In North America, the infection rate for *E. coli* O157:H7 was 0.9 per 100,000 in 2004. In Asia, Japan has experienced the most problems related to *E. coli* O157:H7 (2.74 per 100,000 averaged between 1999 and 2004; Duffy et al., 2006). An estimated 73,500 cases of illness, 2000 hospitalisations and 60 deaths occur each year in the USA due to *E. coli* O157 infection (Mead et al., 1999), costing approximately $1 billion a year in medical costs and lost productivity (Wilks et al., 2005). *E. coli* O157:H7 cases in England and Wales have fluctuated somewhat over the last ten years (HPA, 2012; Figure.1**).**

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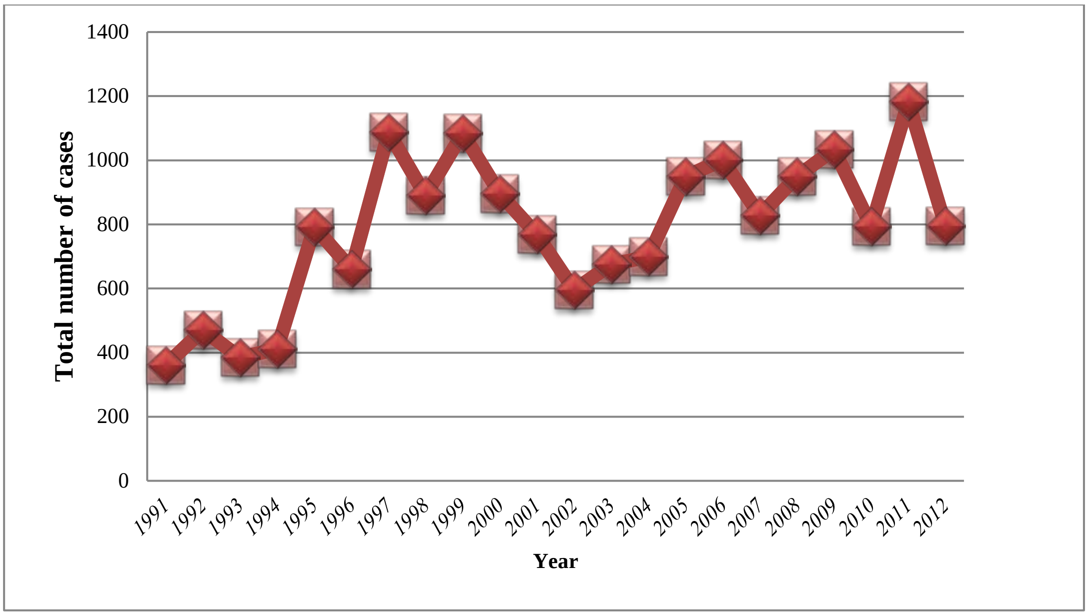

Figure 1. Annual totals of VTEC (*E. coli* O157:H7) infections in England & Wales (HPA, August 2013).

4 Sources of infection

E. coli O157:H7 infections have been associated with a variety of sources and routes. Apart from animal-to-person and person-to-person transmission, consumption of *E. coli* O157:H7-contaminated food, particularly in public places such as day care centres, is an important mode of transmission that has attracted much attention in recent years (Chang & Fang, 2007; Duffy, et al., 2006; EFS, 2007; Liu et al., 2009; Meyer-Broseta et al., 2001). To date, research has investigated survival of the pathogen in a wide range of foods, including meat and meat products (Hwang et al., 2009; Rhoades et al., 2009), dairy products (Voitoux et al., 2002), lettuce (Koseki et al., 2004), apples (Du et al., 2003), tomatoes (Eribo & Ashenafi, 2003), chocolate and other confectionery (Baylis et al., 2004), and drinking water (Schets et al., 2005).

Farm livestock, particularly ruminants like cattle, sheep, and goats, are regarded as the primary reservoirs for VTEC (Heuvelink et al., 1998). Numerous studies have investigated *E. coli* O157:H7 prevalence, transmission, survival and control in cattle and beef (Duffy et al., 2006; Rhoades et al., 2009). Among others, the review by Rhoades et al. (2009) discuss factors that influence the prevalence of three important pathogens, *E. coli* O157:H, *Salmonella enterica*, and *Listeria monocytogenes* in the whole process of meat production. It is estimated that the most severe cases of food-borne disease have been reported to be attributable to various foodstuffs containing beef. For instance, Adak et al. (2005) indicated that in England and Wales, 7% of the 1.7 million cases of food-borne disease in the period 1996-2000, including 67 deaths, were associated with beef. In the Netherlands, undercooked ground beef and raw milk have most often been implicated in food-borne infections (Heuvelink et al., 1998). Different countries may present different situations of food-borne disease, depending on factors such as the pathogen load in the beef products consumed and

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the cooking and consumption habits of the country concerned (Rhoades et al., 2009). Products such as lightly-cooked burgers may be eaten more frequently in the USA, while people in France and the Netherlands consume more steak tartare than people in the UK and Greece.

4.1 Meat

E. coli O157:H7 exists as a normal coloniser of the gastrointestinal tract of cattle (Heuvelink et al., 1998; Nastasijevic et al., 2009). A number of studies have focussed on the prevalence of *E. coli* in the meat chain starting from the farm, the slaughterhouse, to the final, ready-to-eat products (Nastasijevic et al., 2009, Rhoades et al., 2009). The spread of *E. coli* O157:H7 has been identified in farm housing and faeces (Jones, 1999) and the pathogen is known to survive for considerable periods in faeces and slurry (Avery et al., 2004). This bacterium may readily leach from sheep and cattle faeces during rainstorm events thus leading to further infections (Williams et al., 2005). Pigs and poultry can also be a source of O157 VTEC strain. Heuvelink et al. (1999) found that *E. coli* O157:H7 strains were isolated from 1.4% of 145 pigs and from 1.3% of 459 pooled faecal samples from turkey flocks but was negative in faecal samples from chicken flocks. In a similar study, Kijima-Tanaka et al. (2005) isolated *E. coli* O157:H7 from 23% of 62 bovine faecal samples and 14% of 25 swine samples and again there was no isolation from chicken samples. A Korean study by Jo et al. (2004) reported a higher prevalence of *E. coli* O157:H7 in cattle than in pigs (8.4% versus 0.3%) and none in chicken. At slaughter, transmission of *E. coli* from faecal material and hides to carcasses varies from 4.5% to 56% and from 1.1% to 43.4% respectively, which poses a great threat for the contamination of raw meat with this pathogen (Nastasijevic et al., 2009). In addition, contamination may occur during the dressing, skinning and evisceration phases (Nastasijevic et al., 2009). Reinstein et al. (2009) examined the prevalence of *E. coli* O157:H7 in organically and conventionally raised beef cattle at slaughter and found 14.8% and 14.2% positives, respectively. An Irish study recovered *E. coli* O157 from 2.4% of beef trimmings samples, 3.0% of beef carcasses and 3.0% of head meat samples (Carney et al., 2006). The probability of *E. coli* O157:H7 spreading during the mincing process may be highest in the meat chain (Hawker et al., 2001). One carcass contaminated with *E. coli* O157:H7 may quickly spread the bacterium across the whole batch of minced meat from uninfected cows. Cagney et al. (2004) detected *E. coli* O157:H7 in 2.8% of minced beef and beef burgers, both frozen and fresh, in the Republic of Ireland. Magwira et al. (2005) investigated 400 meat samples (134 meat cubes, 133 minced meats, 133 fresh sausages) collected from 15 supermarkets and butcheries in Botswana and found prevalence rates of *E. coli* O157:H7 were 5.2 % in meat cube samples, 3.8 % in minced meat samples, and 2.3 % in fresh sausages. In South Africa, the prevalence of *E. coli* O157:H7 was identified on selected meat and meat products (45 samples each of biltong, cold meat, mincemeat, and polony) (Abong'o & Momba 2009). Strains of *E. coli* O157:H7 were isolated by enrichment culture and confirmed by polymerase chain reaction (PCR). Also investigated were the arteriogram profiles of the *E. coli* O157:H7 isolates. Five (2.8%) out of 180 meat and meat products examined were positive for *E. coli* O157:H7. A parallel study in Switzerland (Fantelli & Stephan, 2001) was conducted on minced meat (beef and pork) samples to test for the presence of STEC. STEC was isolated from 2.3% minced beef samples and 1% minced pork samples.

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4.2 Dairy products

Outbreaks of *E coli* O157:H7 illness have been found to be linked with consumption of raw milk and cheeses made from unpasteurized milk (Elhadiay and Mohammed, 2012; Vernozy-Rozand, 2005). In 1999, more than 11% of the total number of *E. coli* O157:H7 infections in England and Wales were due to unpasteurized milk and dairy product (CDSC, 2000, cited in Vernozy-Rozand, 2005); most probably due to faecal contamination during milking (Hussein & Sakuma, 2005). Conedera et al. (2004) noted that although the prevalence of *E. coli* O157:H7 in raw milk and cheese is low, the organism appears to be able to survive the various stages of the cheese-making process. They found that the heat treatment of milk at the beginning stages of cheese production is usually not sufficient to kill the contaminated vegetative bacteria which can later survive the manufacturing and curing procedures. For example, *E. coli* O157:H7 was found to survive during the manufacturing process of soft Hispanic-type cheese (Kasrazadeh & Genigeorgis, 1995). *E. coli* O157:H7 is characterized by its ability to survive in acidic environments (e.g. in cheddar cheese after a curing period of more than two months; Reitsma & Henning, 1996). Furthermore, fermented dairy products made from raw milk contaminated with *E. coli* O157 can pose a risk to human health (Vernozy-Rozand et al., 2005). Marek et al. (2004) examined the survival of *E. coli* O157:H7 in pasteurized and unpasteurized Cheddar cheese whey. Five strains of *E. coli* O157:H7 were used for the study and were inoculated into 100 ml of fresh, pasteurized or unpasteurized Cheddar cheese whey at 10^5 or 10^2 CFU ml^{-1}, and stored at varying temperatures. Results showed that survival of *E. coli* O157:H7 was significantly higher in the pasteurized whey compared to that in the unpasteurized samples at all storage temperatures. Stringent sanitary practices should therefore be undertaken, particularly during the storage and handling of whey and use of pasteurized milk for cheese manufacture.

4.3 Vegetables and fruits

In the past ten years, an increased number of *E. coli* O157:H7-related outbreaks have been associated with fresh produce such as lettuce, cantaloupe, and alfalfa sprouts (Doyle & Erickson, 2008; Silagyi et al., 2009; Pathanibul et al., 2009). This growing tendency could be due to increased consumptions of potentially risky fresh-cut pre-packaged products (Doyle & Erickson, 2008). Four separate outbreaks of food-borne *E. coli* O157 infections were recorded in USA in 2006 (Doyle & Erickson, 2008). Common vehicles of the disease noted were fruits and vegetables such as green-based salads, potatoes, lettuce, unspecified fruits, and sprouts (Doyle & Erickson, 2008). Among the reported outbreaks, lettuce was the single most frequently mentioned produce (Ackers et al., 1998; López-Gálvez et al., 2009). Ackers et al. (1998) found 70% of patients in 40 Montana residents were infected with *E. coli* O157:H7 due to the consumption of purchased leaf lettuce. In addition, Eribo and Ashenafi (2003) demonstrated that *E. coli* O157:H7 could be found in tomato and processed tomato products as well as products containing vinegar. *E. coli* O157:H7 showed the ability to grow during germination a sprouting of alfalfa (Castro-Rosas & Escartin, 2008) and in acidic foods such as fermented Spanish-style table olives (Spyropoulou et al., 2001).

5 Resilience of *E. coli* O15:H7 to environmental conditions

Environmental conditions such as temperature, pH value, water activity, and sodium chloride have important implications in the survival and growth rates of *E. coli* O157:H7 in foodstuffs.

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The bacterium is known to have a typical resistance to heat (Kaur et al., 1998). It can proliferate at a temperature range of 8-44.5 °C, with the optimal temperature for growth at 37 °C (Edwards & Fung, 2006). Cooking beef thoroughly to 71 °C is effective in eliminating the organism (Doyle & Schoeni, 1984); although slow cooking of meats may not eradicate the organism as well as rapid heating (Edwards & Fung, 2006; Kaur et al., 1998). Regardless of pH and water activity, survival of *E. coli* was found to be better at 5 °C than at 20 or 30 °C in tryptic soy broth (TSB) (Rocelle et al., 1996). Moreover, *E. coli* O157:H7 was found to survive but not grow during fermentation, drying, or subsequent storage at 4 °C for 2 months (Glass et al., 1992). The heat-resistant property of *E. coli* O157:H7 is relative as it can be influenced by many other environmental factors, including growth phase, the amount of heat applied, the rate of heating and the water activity (Kaur et al., 1998). For instance, at 30 °C, inhibition of growth of *E. coli* O157:H7 in TSB was enhanced by reduction of the water activity (Rocelle et al., 1996) as well as increase of sodium chloride concentration (Jordan and Davies, 2001).

Much evidence has shown that pH value plays a primary role in the growth rates of *E. coli* O157:H7. For instance, growth rates are similar at moderate pH values (pH 5.5-7.5), but decrease significantly at lower pH values (Edwards & Fung, 2006). Yet, Benjamin & Datta (1995) found the organism to be acid tolerant under the optimal temperature (37 °C), surviving at pH 2.5 for up to 7 h. The pathogen is capable of acid-adaption and adapted cells have shown increased survival in shredded dry salami and apple cider (Leyer et al., 1995). *E. coli* O157:H7 has been reported to survive for months in acidic foods, such as fermented sausages (CDC, 1995) and apple cider and apple juice (Du et al., 2003); even though products such as fermented sausage may also lead to water stress in bacteria. The resilience of the organism to a combination of factors such as temperature, pH, water activity and sodium chloride can all contribute to the survival and growth of *E. coli* O157:H7 in foodstuffs. Its ability to withstand low pH environments is also of course crucial during passage through the gastro-intestinal tract of livestock and humans.

E. coli O157 can survive and grow in both aerobic and anaerobic conditions as well as modified atmospheres used for food packaging (Bromberg et al., 1998). As a facultative anaerobe, the heat resistance of this pathogen can vary between anaerobic and aerobic environments. For instance, it has been documented that there was little influence on the capability of *E. coli* O157:H7 under anaerobic conditions, but when aerobically-situated, the pathogen showed reduced heat-resistance (Bromberg et al., 1998). Consequently, this has important implications in food packing. Therefore, there may be increased risk of *E. coli* O157:H7 surviving during heating treatments of foodstuffs that are packed under vacuum or reduced oxygen atmospheres (George et al., 1998).

6 Control of *E. coli* O157:H7

The increase in number of food-borne pathogenic infections has generated considerable efforts in the control of organism such as *E. coli* O157 in food. Many preventative measures have been introduced and targeted at all stages of the food chain, from the farm, to the slaughterhouse, and to the preparation of food at home (Vernozy-Rozand et al., 2002; Zhu et al., 2009).

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Although total elimination of *E. coli* O157:H7 carriage in livestock appears unlikely, pathogen transmission can be reduced through a number of farm management practices, such as to prohibit farmers from applying slurry and animal manure to vegetables and fruit plants (Jones, 1999). Good hygiene practices such as careful preparation and cooking of food and interventions such as pasteurization, organic acid washes, and stream vacuuming, as well as the use of antimicrobial solutions (e.g. dilute lactic acid, trisodium phosphate and chlorine) can be effective means to eliminate *E. coli* O157:H7 from food (Marshall et al., 2005; Rhoades et al., 2009; Vernozy-Rozand et al., 2002). For instance, to prevent minced meat from contamination with *E. coli* O157:H7 during the mincing process, cooking at a high temperature can destroy *E. coli* O157:H7 cells (Abong'o & Momba, 2009). To prevent contamination of apple cider, it is suggested to wash and brush apples and preserve the cider with sodium benzoate (Zhao et al., 1993; cited in Chapman, 1995) or aqueous commercial cleaner (Kenney & Beuchat, 2002). To reduce the number of VETC on salad vegetables, storing salad vegetables at 4 °C can be an effective means (Abdul-Raouf et al., 1993).

In recent years, advanced technologies have also been explored in the produce industry to reduce *E. coli* O157:H7and other pathogens as well as to maintain the sensory quality of the produce itself (Arqués et al., 2015). In 2007, Muthukumarasamy and Holley investigated the effect of probiotic incorporation in dry fermented sausages before and after they were micro-encapsulated on the viability of *E. coli* O157:H7. They found that there is a reduction in the viability of *E. coli* O157:H7. On the other hand, they reported that micro-encapsulation increased survival of probiotic strains, maintained sensory properties but reduced their inhibitory action against *E. coli* O157:H7. One study by Selma et al. (2008) showed the combined application of gaseous ozone and hot water could effectively control microbial growth in cantaloupe melon as well as maintain its initial sensory quality such as aroma and texture. However, this study failed to point out specific action of ozone in inactivating *E. coli* O157:H7. Mahmoud (2010) explicitly demonstrated the efficacy of X-ray on inoculated *E. coli* O157:H7 (also including *L. monocytogences*, *S. enterica* and *S. flexneri*) on shredded iceberg lettuce. By treating iceberg lettuce with 1.0 and 2.0 KGy X-ray, the study detected significant reductions of *E. coli* O157:H7 population in both conditions. This approach also showed its promising application because the sensory quality (i.e., visual colour) of leaves was not adversely affected during subsequent storage. Recently, the development of multistrain probiotic dairy products with good technological properties, has gained increased interest as protective cultures against infections (Arqués et al., 2015).Although controlling *E. coli* O157:H7 in food through thermal treatment, chemical destruction and preventative interventions have showed some due efficacy, some studies also report negative findings. For instance, organic acids such as lactic acid and citric acid were reported ineffective in controlling *E. coli* O157:H7 in beef burgers, even when combined with freezing at -20 °C for 2 hours (Bolton et al., 2002). Another study on traditional Iranian barbecued chicken (TIBC) reported that although essential oils of oregano and nutmeg showed effectiveness in inhibiting the growth of *E. coli* O157 H:7 in a broth culture system, they reported no inhibitory effect against this pathogen in ready-to-cook TIBC, suggesting that *in vitro* investigation may not necessarily be applicable to food conditions (Shekarforoush et al., 2007). Although the importance of temperature control and protective packaging has been emphasized in reducing pathogen growth on raw meat, inoculated *E. coli* O157:H7 strain NCTC 12900 could still increase when lamb chops were kept at 4 °C for 12 days (Barrera et al., 2007).

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Elevated public concerns about the adverse consequences of chemically synthesized preservatives used in food industry have diverted research to the application of natural antimicrobials to inhibit *E. coli* O157:H7 growth and activity (Raybaudi–Massilia et al., 2009). Numerous studies have been done using antimicrobials of animal, plant and microbial origin to prevent or control microbial contamination in fresh–cut fruits and fruits juice (Raybaudi–Massilia et al., 2009). For instance, essential oils containing thymol, eugenol, and carvacrol can disrupt the cellular membrane and cell walls of microorganisms such as gram–negative/positive (Raybaudi–Massilia et al., 2009). Cinnamic aldehyde extracted from cinnamon shoot also showed antimicrobial activity and was significant in reducing *E. coli* O157:H7 number at 37 °C after 12 hours (Kim et al., 2004).

Considering the safe and quality of food when antimicrobials are applied to preventing pathogenic contamination, recent years have witnessed the emerging use of chitosan in food industry due to its antimicrobial activity against a wide range of food–borne bacteria (No et al., 2007; Goy et al., 2009). In particular, chitosan has attracted much attention, used as antimicrobial packaging material to assist the preservation of perishable foods and extend their shelf life (Lee et al., 2003).

7 Conclusion

Escherichia coli O157 is an important food–borne bacterial pathogen closely associated with many severe human illnesses such as haemolytic uremic syndrome. Many of the intervention measures described are still effective at experimental stage and are unlikely to be widely implemented in the foreseeable future due to a lack of commercial viability, geographical differences in the regulatory framework, or a lack of acceptance by consumers. Elevated public concerns about the adverse consequences of chemically synthesized preservatives used in food industry have diverted research to the application of natural antimicrobials to inhibit *E. coli* O157:H7 growth and activity.

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Thermohaline convection in a horizontal porous layer affected by a non-linear magnetic field

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Abstract

This work examines the thermohaline convection of an infinite horizontal porous layer permeated by an incompressible thermally and electrically conducting viscous solutal fluid and affected by a non-linear vertical magnetic field. A non-linear constitutive relationship between the magnetic field and the magnetic induction has been used. This non-linear relationship has been proposed by Roberts [13] in the context of neutron stars, but the results obtained are also relevant to the area of ferromagnetic fluids. The non-linear relationship has no effect on the development of instabilities through the mechanism of stationary convection, but influences the threshold of overstable convection which is often the preferred mechanism in non-terrestrial applications.

The effects of solute concentration, magnetic field and porosity have been discussed. Numerical results were obtained using the method of expansion of Chebyshev polynomials for the case when the fluid is heated from below and the solute concentration increases upwards for different boundary conditions.

Keywords: Benard convection, magnetic field, Porous layer, Thermohaline convection.

1 Introduction

The stability of a thermohaline conducting porous layer permeated by an incompressible thermally and electrically conducting viscous solutal fluid is a subject of interest for its practical applications in geophysics, soil sciences and ground water hydrology. In such problems buoyancy forces can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentrations. This subject has received great importance and interests for the last decades (Walin, 1964; Veronis, 1965; Nield, 1967; Murray and Chen, 1989; Khare and Sahai 1992, 1993; Abdullah, 2000; Bahloul et al., 2003; Hayat et al., 2007; Wang and Tan, 2009; Sekar and Raju, 2014; Luo and Gong, 2014). The thermohaline instability of a heterogenous fluid in porous medium using Brinkman model is studied by Khare and Sahai (1992). This problem has been extended by Khare and Sahai (1993) to include the effect of magnetic field.

In (Khare and Sahai 1992) a linear constitutive relationship between the magnetic field $\boldsymbol{H}$ and the magnetic induction $\boldsymbol{B}$ is used. However Muzikar and Pethick (1981) suggested that a non-linear relationship between the magnetic field and the magnetic induction may be appropriate for certain classes of materials relevant to neutron stars. Abdullah and Lindsay (1990, 1991) used this non-linear relationship to discuss the instability of the magnetic Benard problem for a vertical and non-vertical magnetic field respectively. This non-linear relationship is also applied by Abdullah (2000) to study the thermosolutal convection in a nonlinear magnetic fluid.

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In this paper we shall study, the thermohaline convection instability in a horizontal porous layer permeated by an incompressible, thermally and electrically conducting fluid using Brinkman model in the presence of a non-linear relationship between magnetic field and magnetic induction suggested by Roberts (1981) for the cases of both stationary and overstability. Analytical solutions shall be obtained when both boundaries are free and numerical results will be presented for the cases of free and rigid boundaries. The numerical computations shall be obtained using a D^2 Chebyshev tau method which is highly useful in obtaining accurate eigenvalues for one and two layers problems. This method has been extensively applied by many authors (Straughan, 2002; Car, 2004; Chang et al. 2006; Hill and Straughan, 2008; Shivakumara, 2011; Banjer and Abdullah, 2012; Chandrasekar 1981).

2 Mathematical formulation

Consider an infinite horizontal layer occupied by a porous medium permeated by an incompressible, thermally and electrically conducting viscous soluted fluid of density ρ. The fluid is subject to a constant gravitational acceleration in the negative x3 direction. A constant magnetic field is imposed across the layer in the positive x3 direction which has the representation **B** = (0,0, *B*).

In order to fully describe the nature of this model we need to discuss the interaction between electromagnetic and mechanical effects and so we define **E**, and **J** to be respectively the electric field and the current density. The magnetic variables are required to satisfy the Maxwell's equations

$$Div\,\mathbf{B} = B_{i,i} = 0,$$

$$(Curl\,\mathbf{H})_i = e_{ijk} H_{k,i} = J_i\,, \tag{1}$$

$$(Curl\,\mathbf{E})_i = e_{ijk} E_{k,j} = -\frac{\partial B_i}{\partial t},$$

where the displacement current has been neglected as is customary in this type of problems and where the current density **J** is given by

$$\eta\, J_i = E_i + e_{ijk}\upsilon_j B_k\,, \tag{2}$$

where υ_j is the fluid velocity and η is the resistivity (assumed constant). In this problem we shall use the non-linear relation between **B** and **H** of the form (see [13])

$$H_i = \rho\phi B_i \tag{3}$$

where $B = \sqrt{(B_i B_i)}$ is the magnitude of the magnetic induction and ϕ is related to the partial derivative of the magnetic free energy with respect to *B*. The conventional magnetic permeability, μ, is given by

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$$\mu(\rho, B) = (\rho\phi)^{-1}.$$

The strength of the non-linear permeability is measured in terms of the non-dimensional magnetic number ε where

$$\varepsilon = -\frac{B}{\mu(\rho, B)}\frac{\partial\mu}{\partial B}.$$

Conventional ideas would indicate that the permeability is a decreasing function of B and so $\varepsilon \geq 0$. When $\varepsilon = 0$ then the magnetic permeability μ is constant and this area has been extensively researched by [24]. If the Boussinesq approximation is used then the governing equations are

$$\begin{aligned}
\dot{\upsilon}_i &= -(P/\rho_0)_{,i} + \nu\nabla^2\upsilon_i - g(1-\alpha\theta+\alpha' C)\delta_{i3} + (\phi B_i)_{,k} - \frac{\nu}{k_1}\upsilon_i,\\
\upsilon_{i,i} &= 0,\\
\dot{\theta} &= \kappa\,\nabla^2\theta,\\
\dot{C} &= \kappa'\,\nabla^2 C,\\
B_{i,i} &= 0,\\
J_i &= e_{ijk}H_{k,j},\\
\frac{\partial B_i}{\partial t} &= \upsilon_{i,j}B_j - \upsilon_j B_{i,j} - \eta e_{ijk}J_{k,j},
\end{aligned} \tag{4}$$

where ν is the kinematic viscosity, g is acceleration due to gravity, θ is absolute temperature, C is solute mass concentration, α, α' are the coefficients of volume and solvent expansion, k_1 is the permeability of porous medium and κ and κ' are the coefficients of thermal and soluted diffusivity and $P = p + \frac{\rho}{2}(B^2\phi + \int B^2\phi_B\, dB)$ is the modified pressure. We may observe that equations (4) have a steady state solution in which

$$\begin{aligned}
&\boldsymbol{\upsilon} = 0, \quad \boldsymbol{J} = 0, \quad \theta = \theta(x_3) = T_0 - \beta x_3, \quad \beta = \frac{\tilde{T}}{d},\\
&C = C(x_3) = S_0 - \beta' x_3, \quad \beta' = \frac{\tilde{S}}{d},\\
&P = P(x_3), \quad \boldsymbol{B} = (0,0,B), \quad B = \text{constant}, \quad \phi = \phi(B)
\end{aligned} \tag{5}$$

where β is the adverse temperature gradient and β' is the adverse concentration gradient. The temperatures on the planes $x_3 = 0$ and $x_3 = d$ are T_0 and $T_0 - \tilde{T}$ respectively and the solute

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concentrations on the planes $x_3 = 0$ and $x_3 = d$ are S_0 and $S_0 - \tilde{S}$ respectively. Although we consider the case in which the external magnetic field is normal to the layer of the fluid, a similar analysis can be applied to any constant $\boldsymbol{B}$.

3 The perturbation equations

We shall assume that the steady solution (5) be perturbed by the following linear perturbation quantities

$$\boldsymbol{\upsilon} = 0 + \delta\ \hat{\boldsymbol{\upsilon}}, \qquad \boldsymbol{J} = 0 + \delta\ \hat{\boldsymbol{J}}, \qquad \theta = \theta + \delta\ \hat{\theta}, \qquad C = C + \delta\ \hat{C},$$

$$P = P + \delta\ \hat{P}, \qquad \boldsymbol{B} = (0,0,B) + \ \delta\, \boldsymbol{b}, \qquad \phi = \phi + \delta\, \hat{b}_3\, \phi_B$$

where $\hat{\boldsymbol{\upsilon}}, \hat{\boldsymbol{J}}, \widehat{\theta}, \widehat{C}, \widehat{P}$ and $\hat{b}$ are respectively the linear perturbations of velocity, current density, temperature, solute concentration, pressure and magnetic induction, about their values described in (5). The linear perturbation of ϕ about its value is given by

$$\phi \approx \phi(B) + \delta\ \hat{b}_3 \phi_{,B}(B), \qquad \hat{b} = (\hat{b}_1, \hat{b}_2, \hat{b}_3).$$

We may verify that the linearized version of (4) are

$$\begin{aligned}
&\frac{\partial \hat{\upsilon}_i}{\partial t} = -(\hat{P}/\rho_0)_{,i} + \nu\nabla^2\hat{\upsilon}_i + g(\alpha\hat{\theta} - \alpha'\hat{C})\delta_{i3} + B\phi\hat{b}_{i,3} + B^2\phi_B\hat{b}_{3,3}\delta_{i3} - \frac{\nu}{k_1}\hat{\upsilon}_i, \\
&\hat{\upsilon}_{i,i} = 0, \\
&\frac{\partial \hat{\theta}}{\partial t} - \beta\hat{\upsilon}_3 = \kappa\ \nabla^2\hat{\theta}, \\
&\frac{\partial \hat{C}}{\partial t} - \beta'\upsilon_3 = \kappa'\ \nabla^2\hat{C}, \\
&\hat{b}_{i,i} = 0, \\
&\hat{J}_i = e_{ijk}(\rho\phi\hat{b}_k + \rho B\phi_B\hat{b}_3\delta_{k3})_{,j}, \\
&\frac{\partial \hat{b}_i}{\partial t} = B\hat{\upsilon}_{i,3} - \eta\ e_{ijk}\hat{J}_{k,j}.
\end{aligned} \tag{6}$$

At this stage we introduce the dimensionless variables $x_i^*, t^*, v_i^*, J_i^*, C^*, \theta^*, P^*$ and b_i^* such that

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$$x_i = dx_i^* \qquad t = \frac{d^2}{\nu} t^* \qquad \hat{\upsilon}_i = \frac{\kappa}{d}\upsilon_i^* \qquad \hat{J}_i = \frac{\kappa\nu\rho_0}{d^3 B} J_i^* \qquad \hat{C} = \frac{\kappa}{d}\sqrt{\frac{\nu|\beta'|}{\kappa'\alpha' g}}\, \mathrm{C}^*$$

$$\hat{\theta} = \frac{\kappa}{d}\sqrt{\frac{\nu|\beta|}{\kappa\alpha g}}\,\theta^* \qquad \hat{P} = \frac{\rho_0\kappa\nu}{d^2} P^* \qquad \hat{b}_i = \frac{\kappa\nu}{B\phi d^2} b_i^*$$

After this non-dimensionalization, (6) are simplified to

$$\frac{\partial\upsilon_i}{\partial t} = -P_{,i} + \nabla^2\upsilon_i + \sqrt{R_t}\;\theta\;\delta_{i3} - \sqrt{R_s}\;C\delta_{i3} + b_{i,3} + \varepsilon b_{3,3}\delta_{i3} - \frac{1}{N}\upsilon_i\,,$$

$$\upsilon_{i,i} = 0\,,$$

$$P_r\frac{\partial\theta}{\partial t} + M\sqrt{R_t}\,\upsilon_3 = \nabla^2\theta\,,$$

$$P_r'\frac{\partial C}{\partial t} + M'\sqrt{R_s}\,\upsilon_3 = \nabla^2 C\,, \tag{7}$$

$$b_{i,i} = 0\,,$$

$$J_i = e_{ijk}(b_{k,j} + \varepsilon b_{3,j}\delta_{k3})\,,$$

$$P_m\frac{\partial b_i}{\partial t} = Q\upsilon_{i,3} - e_{ijk}J_{k,j}\,,$$

where the ($*$) superscript has been dropped but all the variables are now non-dimensional and where the non-dimensional numbers $R_t, R_s, N, P_r, P_r', P_m, \varepsilon$ and Q are given by

$$R_t = \frac{\alpha g|\beta|}{\kappa\nu}d^4 \qquad R_s = \frac{\alpha' g|\beta'|}{\kappa'\nu}d^4 \qquad N = \frac{k_1}{d^2} \qquad P_r = \frac{\nu}{\kappa}$$

$$P_r' = \frac{\nu}{\kappa'} \qquad P_m = \frac{\mu\nu}{\eta} \qquad \varepsilon = \frac{B}{\phi}\phi_B \qquad Q = \frac{B^2 d^2}{\rho_0\nu\eta}$$

and where

$$M = -\frac{\beta}{|\beta|} = \begin{cases} 1 & \text{when heating from above,} \\ -1 & \text{when heating from below.} \end{cases}$$

$$M' = -\frac{\beta'}{|\beta'|} = \begin{cases} 1 & \text{when solute concentration increases upwards,} \\ \text{-1} & \text{when solute concentration decreases upwards.} \end{cases}$$

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4 The boundary conditions

The fluid is confined between the planes $x_3 = 0$ x_3 and $x_3 = d$ and on these planes, we need to specify mechanical, thermal, solutal and electromagnetic conditions. For a free surface, the conditions are

$$v_3 = 0, \qquad \frac{\partial^2 v_3}{\partial x_3^2} = 0, \qquad \frac{\partial \xi_3}{\partial x_3}$$

where ξ_3 is the normal component of the vorticity. For a rigid surface, the conditions are

$$v_3 = 0, \qquad \frac{\partial v_3}{\partial x_3} = 0, \qquad \xi_3 = 0.$$

The thermal conditions are

$$\theta = \theta_{ext} \qquad \text{on a conducting boundary,}$$

$$\frac{\partial \theta}{\partial x_3} = 0 \qquad \text{on an insulating boundary}$$

where θ_{ext} is the temperature of the region exterior to the fluid boundary. The solute conditions are

$$C = C_{ext} \qquad \text{on a permeable boundary,}$$

$$\frac{\partial C}{\partial x_3} = 0 \qquad \text{on an impermeable boundary}$$

where C_{ext} is the solute concentration in the region exterior to the boundary. On a perfectly insulating electromagnetic boundary, no current can flow to the exterior region and the magnetic field is continuous across the boundary with the external magnetic field being derived from a scalar potential since curl $\boldsymbol{H} = 0$ in the exterior region. On a stationary perfectly conducting boundary, the surface components of electric field are zero as is the time derivative of the normal component of the magnetic induction. It is common practice to associate mechanically rigid and electrically perfectly conducting stationary boundaries. In this case the surface components of current density are also zero and so the electromagnetic boundary conditions assume the form

$$\frac{\partial b_3}{\partial t} = 0 \qquad \frac{\partial J_3}{\partial x_3} = 0$$

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5 The eigenvalue problem

We aim to investigate the linear stability of the steady solution (5) and with this aim in mind we construct the related eigenvalue problem from (7) and the boundary conditions. As is the case in many convection problems, the vector components parallel to the direction of gravity play a central role and so it is convenient to introduce the variables w, b, J, ξ and z by the definitions

$$w = v_3, \quad b = b_3, \quad J = J_3, \quad \xi = \xi_3, \quad z = x_3.$$

Now we look for a solution of the form

$$\psi = \psi(z)\exp\left[i(nx+my)+\sigma t\right]$$

where n, m are the wave numbers of the harmonic disturbance and σ is the growth rate. Thus the relative equations become

$$\sigma\xi = L\xi + DJ - \frac{1}{N}\xi \ ,$$

$$\sigma P_m J = QD\xi + LJ \ ,$$

$$\sigma Lw = L^2 w - a^2\sqrt{R_t}\,\theta + a^2\sqrt{R_s}\,C + LDb - \varepsilon a^2(Db) - \frac{1}{N}Lw \ , \tag{8}$$

$$\sigma P_m b = QDw + Lb - \varepsilon a^2 b \ ,$$

$$\sigma P_r\theta = L\theta - M\sqrt{R_t}\,w \ ,$$

$$\sigma P_r' C = LC - M'\sqrt{R_s}\,w \ .$$

Here D is the operator $\frac{\partial}{\partial z}$, a (= $\sqrt{n^2+m^2}$ is the wave number and L is the operator $(D^2 - a^2)$. It is clear from equations (8) that ξ and J are decoupled from w, b and θ and so we can ignore equations $(8)_{1,2}$. We may eliminate b, θ and C from $(8)_3$ using equations $(8)_{4,5,6}$ to obtain

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$$
\begin{aligned}
&\sigma^4 P_m P_r P_r' Lw - \sigma^3 \Big\{ (P_m P_r + P_m P_r' + P_r P_r' + P_m P_r P_r')L^2 w - P_r(\varepsilon a^2 P_r' + \frac{1}{N} P_m P_r')Lw \Big\} \\
&+\sigma^2 \Big\{ (P_m + P_r + P_r' + P_m P_r + P_r P_r' + P_m P_r')L^3 w - \Big[QP_r P_r' + \varepsilon a^2 (P_r + P_r' + P_r P_r') + \frac{1}{N}(P_m P_r + P_m P_r' \\
&+P_r P_r') \Big] L^2 w - P_r P_r' (Qa^2 - \varepsilon a^2 Q - \frac{1}{N}\varepsilon a^2)Lw - a^2 P_m (MP_r' R_t - M' P_r R_s)w + \varepsilon a^4 P_r P_r' Qw \Big\} \\
&-\sigma \Big\{ (1 + P_m + P_r + P_r')L^4 w - \Big[Q(P_r + P_r') + \sigma a^2 (1 + P_r + P_r') + \frac{1}{N}(P_m + P_r + P_r') \Big] L^3 w \\
&+\Big[a^2 (\varepsilon Q - Q + \frac{1}{N}\varepsilon)(P_r + P_r') \Big] L^2 w + \Big[-a^2 MR_t (P_r' + P_m) + a^2 M' R_s (P_m + P_r) + \varepsilon a^4 Q (P_r + P_r') \Big] Lw \\
&+\varepsilon a^4 (MR_t P_r' - M' P_r R_s) w \Big\} + L^5 w - (Q + \varepsilon a^2 + \frac{1}{N})L^4 w - a^2 (Q - \varepsilon Q - \frac{1}{N}\varepsilon)L^3 w + a^2 (R_s M' + \varepsilon a^2 Q \\
&-MR_t)L^2 w + a^4 \varepsilon (MR_t - M' R_s)Lw = 0
\end{aligned}
\tag{9}
$$

which is a tenth order ordinary differential equation to be satisfied by w. In the following analysis we shall consider both boundaries to be free but later on we shall present results for the corresponding rigid boundary value problem. For the free boundary problem

$$w = D^2 w = 0 \quad \text{on} \quad x_3 = 0\ ,\ 1,$$

and if we suppose that $w = A\sin(l\pi z)$, where A is a constant and l is an integer, then

$$Lw = -\lambda w, \qquad \text{where} \quad \lambda = l^2 \pi^2 + a^2$$

and equation (9) becomes

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$$
\begin{aligned}
&\sigma^4 P_m P_r P'_r + \sigma^3 \Big\{ (P_m P_r + P_m P'_r + P_r P'_r + P_m P_r P'_r)\lambda + P_r P'_r \big(\varepsilon a^2 + \frac{1}{N} P_m\big) \Big\} \\
&+\sigma^2 \Big\{ (P_m + P_r + P'_r + P_m P_r + P_r P'_r + P_m P'_r)\lambda^2 + \Big[\varepsilon a^2 (P_r + P'_r + P_r P'_r) + \frac{1}{N}(P_m P_r + P_m P'_r + P_r P'_r) \Big]\lambda \\
&+\frac{1}{N}\varepsilon a^2 P_r P'_r + Q l^2 \pi^2 P_r P'_r (\lambda + \varepsilon a^2)\, \lambda^{-1} + P_m a^2 (M P'_r R_t - M' P_r R_s)\lambda^{-1} \Big\} + \sigma \Big\{ (1 + P_m + P_r + P'_r)\lambda^3 \\
&+\Big[\varepsilon a^2 (1 + P_r + P'_r) + \frac{1}{N}(P_m + P_r + P'_r) \Big]\lambda^2 + Q l^2 \pi^2 (P_r + P'_r)(\lambda + \varepsilon a^2) + \frac{1}{N}\varepsilon a^2 (P_r + P'_r)\lambda \\
&+a^2 \big[M R_t (P_m + P'_r) - M' R_s (P_m + P_r) \big] + \varepsilon a^4 (M R_t P'_r - M' P_r R_s)\lambda^{-1} \Big\} + \lambda^4 + \big(\varepsilon a^2 + \frac{1}{N}\big)\lambda^3 + \frac{1}{N}\varepsilon \lambda^2 \\
&+Q l^2 \pi^2 (\lambda + \varepsilon a^2)\lambda + a^2 (M R_t - M' R_s)\lambda + \varepsilon a^4 (M R_t - M' R_s) = 0.
\end{aligned}
\tag{10}
$$

The solutions of (10) are functions of $P_r, P'_r, P_m, N, \varepsilon, Q, R_t$ and R_s and we have to examine how the nature of these solutions depends on those variables by discussing the following cases.

Case 1. The fluid is heated from above and the solute concentration decreases upwards.

Here $M = 1, M' = -1$. To find the critical Rayleigh number for the onset of stationary convection we set σ=0 in equation (10) to obtain

$$
R_t = -\left[\frac{\lambda}{a^2}\big(\lambda^2 + \frac{1}{N} + Q\pi^2\big) + R_s \right], \quad l^2 = 1
$$

since R_t is negative then no stationary convection happened. To obtain the critical Rayleigh number for the overstability case we suppose that

$$
a^2 = \pi^2 a_1, \; \sigma = i\,\pi^2 \sigma_1, \; Q = \pi^2 Q_1, \; R_t = \pi^4 R_{t1}, \; R_s = \pi^4 R_{s1}, \; N_1 = \pi^2 N, \; l^2 = 1,
$$

where σ is complex and $\sigma_1 \neq 0$. Substitute into equation (10) to obtain

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$$R_{t1} = -\frac{(1+a_1)(1+a_1+i\sigma_1)(1+a_1+i\sigma_1 P_r)}{a_1} - \frac{(1+a_1+i\sigma_1 P_r)}{(1+a_1+i\sigma_1 P_r')} R_{s1}$$

$$- \frac{(1+a_1+\varepsilon a_1)(1+a_1+i\sigma_1 P_r)}{a_1(1+a_1+i\sigma_1 P_m+\varepsilon a_1)} Q_1 - \frac{(1+a_1)(1+a_1+i\sigma_1 P_r)}{a_1}\frac{1}{N_1}. \tag{11}$$

By equating, separately the real and imaginary parts of (11) and eliminating R_{t1}, we obtain $A_2Y^2 + A_1Y + A_0 = 0$, which is a polynomial of order two in $Y(=\sigma_1^2)$ with constant real coefficients, where

$$A_2 = \zeta P_r'^2 P_m^2 \left(\zeta + \frac{P_r}{N_1}\right),$$

$$A_1 = \zeta^2(1+P_r)\left\{P_m^2\zeta^2 + P_r'^2\left[\zeta+\varepsilon(\zeta-1)\right]^2\right\} + (\zeta-1)\zeta\left[P_m^2(P_r-P_r')\right]R_{s1}$$

$$+ P_r'^2\left[\zeta+\varepsilon(\zeta-1)\right]\left[\zeta(P_r-P_m)-\varepsilon(\zeta-1)P_r\right]Q_1$$

$$+ \zeta\left\{P_rP_r'^2\left[\zeta+\varepsilon(\zeta-1)\right]^2 + \zeta^2P_m^2(P_r-P_r')\right\}\frac{1}{N_1},$$

$$A_0 = \zeta^4(1+P_r)\left[\zeta+\varepsilon(\zeta-1)\right]^2 + (\zeta-1)(P_r-P_r')\zeta\left[\zeta+\varepsilon(\zeta-1)\right]^2 R_{s1}$$

$$+ \zeta^2\left[\zeta+\varepsilon(\zeta-1)\right]\left\{(P_r-P_m)\zeta+\left[\varepsilon(\zeta-1)P_r\right]\right\}Q_1$$

$$+ \zeta^3\left[\zeta^2P_r + 2\varepsilon(\zeta-1)\zeta P_r + \varepsilon^2(\zeta-1)^2P_r\right]\frac{1}{N_1},$$

and $\zeta = 1 + a_1$. Consequently,

$$Y = -\frac{A_1 \pm \sqrt{A_1^2 - 4A_2A_0}}{2A_2}.$$

Since σ_1 is real and not equal to zero, then Y is real and greater than zero. A necessary condition for the case of overstability to happen is that $A_0 < 0$ and this can be satisfied if either case of the following is satisfied:

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(i) $P_m > P_r\left[1+\varepsilon(1-\frac{1}{\zeta})\right],\ P'_r > P_r$.

$$Q_1 > \frac{\left[\zeta+\varepsilon(\zeta-1)\right]}{\zeta\left[(P_r-P_m)\zeta+\varepsilon(\zeta-1)P_r\right]}\left[\zeta^3(1+P_r)-(\zeta-1)(P_r-P'_r)R_{s1}+\zeta^2\frac{P_r}{N_1}\right],$$

(ii) $P_m > P_r\left[1+\varepsilon(1-\frac{1}{\zeta})\right],\ P'_r = P_r$.

$$Q_1 > \frac{\left[\zeta+\varepsilon(\zeta-1)\right]}{\left[(P_r-P_m)\zeta+\varepsilon(\zeta-1)P_r\right]}\left[\zeta^3(1+P_r)+\zeta^2\frac{P_r}{N_1}\right], \tag{12}$$

(iii) $P_m > P_r\left[1+\varepsilon(1-\frac{1}{\zeta})\right],\ P'_r < P_r$.

$$Q_1 > \frac{\left[\zeta+\varepsilon(\zeta-1)\right]}{\zeta\left[(P_r-P_m)\zeta+\varepsilon(\zeta-1)P_r\right]}\left[\zeta^3(1+P_r)+(\zeta-1)(P_r-P'_r)R_{s1}+\zeta^2\frac{P_r}{N_1}\right].$$

To study the effects of solute concentration, magnetic field, and permeability of porous medium on the thermal Rayleigh number. We can show from (11) that

$$\frac{dR_{t1}}{dR_{s1}}=-1, \qquad \frac{dR_{t1}}{dQ_1}=-\frac{\zeta}{(\zeta-1)}, \qquad \frac{dR_{t1}}{dN_1}=\frac{\zeta^2}{(\zeta-1)N_1^2}.$$

It is clear that for the overstability case the solute concentration and the magnetic field have a destabilizing effect on the system, but the permeability of porous medium has a stabilizing effect on the system.

Case 2. The fluid is heated from above and the solute concentration increasing upwards.

In this case $M=1$, $M'=1$. To find the thermal Rayleigh number for the onset of stationary convection, we can deduce from equation (10) that

$$R_t=-\frac{\lambda}{a^2}(\lambda^2+\frac{\lambda}{N}+Q\pi^2)+R_s. \tag{13}$$

It is clear that the stationary instability happens when $R_s > \frac{\lambda}{a^2}(\lambda^2+\frac{\lambda}{N}+Q\pi^2)$. To study the effect of the solute concentration, magnetic field, and permeability of porous medium on R_t we obtain from (13),

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$$\frac{dR_t}{dR_s}=1, \qquad \frac{dR_t}{dQ}=-\frac{\lambda\pi^2}{a^2}, \qquad \frac{dR_t}{dN}=\frac{\lambda^2}{a^2N^2}.$$

So it is clear that the solute concentration and the permeability of porous medium have a stabilizing effect on the system, but the magnetic field has a destabilizing effect on the system. The critical value of $R_t (= R_{crit})$is attained by minimizing R_t over the wave number a. For the overstability case we can show from (13) that

$$R_{t1}=-\frac{(1+a_1)(1+a_1+i\sigma_1)(1+a_1+i\sigma_1P_r)}{a_1}+\frac{(1+a_1+i\sigma_1P_r)}{(1+a_1+i\sigma_1P_r')}R_{s1}$$
$$-\frac{(1+a_1+\varepsilon a_1)(1+a_1+i\sigma_1P_r)}{a_1(1+a_1+i\sigma_1P_m+\varepsilon a_1)}Q_1-\frac{(1+a_1)(1+a_1+i\sigma_1P_r)}{a_1}\frac{1}{N_1}, \tag{14}$$

from which we can conclude that overstability occurs if one of the three pre-mentioned cases in (12) is satisfied. From equation (14), we obtain

$$\frac{dR_{t1}}{dR_{s1}}=1, \qquad \frac{dR_{t1}}{dQ_1}=-\frac{\zeta}{(\zeta-1)}, \qquad \frac{dR_{t1}}{dN_1}=\frac{\zeta^2}{(\zeta-1)N_1^2}.$$

Thus the solute concentration and the permeability of porous medium have a stabilizing effect on the system but the magnetic field has a destabilizing effect on the system.

Case 3. The fluid is heated from below and the solute concentration increasing upwards.

In this case $M = -1$, $M' = 1$. So from (10) the thermal Rayleigh number for the onset of stationary convection has the form

$$R_t=\frac{\lambda}{a^2}(\lambda^2+\frac{\lambda}{N}+Q\pi^2)-R_s.$$

It is clear that the stationary convection is possible provided $R_s < \frac{\lambda}{a^2}(\lambda^2+\frac{\lambda}{N}+Q\pi^2)$. Clearly

$$\frac{dR_t}{dR_s}=-1, \qquad \frac{dR_t}{dQ}=\frac{\lambda\pi^2}{a^2}, \qquad \frac{dR_t}{dN}=-\frac{\lambda^2}{a^2N^2}.$$

Thus the solute concentration and the permeability of porous medium have a destabilizing effect on the system but the magnetic field has a stabilizing effect on the system. Following the same procedure as in case one and two we obtain the thermal Rayleigh number for the overstability case,

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$$R_{t1} = \frac{(1+a_1)(1+a_1+i\sigma_1)(1+a_1+i\sigma_1 P_r)}{a_1} - \frac{(1+a_1+i\sigma_1 P_r)}{(1+a_1+i\sigma_1 P_r')} R_{s1}$$

$$+ \frac{(1+a_1+\varepsilon a_1)(1+a_1+i\sigma_1 P_r)}{a_1(1+a_1+i\sigma_1 P_m+\varepsilon a_1)} Q_1 + \frac{(1+a_1)(1+a_1+i\sigma_1 P_r)}{a_1} \frac{1}{N_1}, \qquad (5.14)$$

from which we conclude that overstability occurs if one of the three pre-mentioned cases in (12) is satisfied and we can show that

$$\frac{dR_{t1}}{dR_{s1}} = -1, \qquad \frac{dR_{t1}}{dQ_1} = \frac{\zeta}{(\zeta-1)}, \qquad \frac{dR_{t1}}{dN_1} = -\frac{\zeta^2}{(\zeta-1)N_1^2}.$$

So it is clear that the solute concentration and the permeability of porous medium have a destabilizing effect on the system but the magnetic field has a stabilizing effect on the system.

Case 4. The fluid is heated from below and the solute concentration decreases upwards.

In this case $M = -1, M' = -1$. So from (10), the thermal Rayleigh number for the onset of stationary convection has the form

$$R_t = \frac{\lambda}{a^2}\left(\lambda^2 + \frac{\lambda}{N} + Q\pi^2\right) + R_s$$

from which we obtain

$$\frac{dR_t}{dR_s} = 1, \qquad \frac{dR_t}{dQ} = \frac{\lambda\pi^2}{a^2}, \qquad \frac{dR_t}{dN} = -\frac{\lambda^2}{a^2 N^2}.$$

It is clear that the solute concentration and the magnetic field have a stabilizing effect on the system but the permeability of porous medium has a destabilizing effect on the system. For the overstability case, we can deduce from (10) that

$$R_{t1} = \frac{(1+a_1)(1+a_1+i\sigma_1)(1+a_1+i\sigma_1 P_r)}{a_1} + \frac{(1+a_1+i\sigma_1 P_r)}{(1+a_1+i\sigma_1 P_r')} R_{s1}$$

$$+ \frac{(1+a_1+\varepsilon a_1)(1+a_1+i\sigma_1 P_r)}{a_1(1+a_1+i\sigma_1 P_m+\varepsilon a_1)} Q_1 + \frac{(1+a_1)(1+a_1+i\sigma_1 P_r)}{a_1} \frac{1}{N_1}, \qquad (5.17)$$

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from which it can be shown that overstability occurs if one of the three pre-mentioned cases in (12) is satisfied. Following the same procedure we can show that

$$\frac{dR_{t1}}{dR_{s1}}=1, \qquad \frac{dR_{t1}}{dQ_1}=\frac{\zeta}{(\zeta-1)}, \qquad \frac{dR_{t1}}{dN_1}=-\frac{\zeta^2}{(\zeta-1)N_1^2}.$$

So it's clear that the solute concentration and the magnetic field have a stabilizing effect on the system but the permeability of porous medium has a destabilizing effect on the system.

6 Results and discussion

In the previous section, several cases have been discussed in the context of heating and solute concentrating of the fluid from above or below. It has been shown that for all cases, the thermal and solute Rayleigh numbers are independent of the strength of the nonlinearity, ε, for stationary convection case. However, this nonlinearity has a profound effect on the development of instabilities through overstable mode for all cases. When the fluid is heated from above and soluted from below (case 1) no instability ensues through stationary convection but we may have overstability under some conditions. In this case the solute concentration and the magnetic field have a destabilizing effect but the permeability of porous medium has a stabilizing effect. When the fluid is heated and soluted from above (case 2) we could have both stationary convection and overstability, and in this case the solute concentration and the permeability of porous medium have a stabilizing effect but the magnetic field has a destabilizing effect. When the fluid is heated from below and soluted from above (case 3) stationary stability and overstability are possible and in this case the magnetic field has a stabilizing effect but the solute concentration and the permeability of porous medium have a destabilizing effect on the system. Finally, when the fluid is heated and soluted from below (case 4) both stationary convection and overstability are possible and both the solute concentration and magnetic field have a stabilizing effect on the system but the permeability of porous medium has a destabilizing effect.

The system of (8) and the boundary conditions constitute a tenth order eigenvalue problem. This system is solved numerically using the method of expansion of Chebyshev polynomials when the fluid is heated from below and soluted from above (case 3). The critical thermal Rayleigh numbers and the critical solute Rayleigh numbers are obtained by minimizing over the wave number for various values of the parameters $P_r, P_r', P_m, N, \varepsilon, Q$. According to the boundary conditions of the solute concentration we have three cases to consider:

Case (1): $DC = 0$ on $x_3 = 0, x_3 = d$.

Case (2): $DC = 0$ on $x_3 = 0$, C = 0 on $x_3 = d$.

Case (3): $C = 0$ on $x_3 = 0, x_3 = d$.

The relations between the solute Rayleigh number, R_s, and the critical thermal Rayleigh number, R_t, for the overstability case for the three cases of solute boundary conditions when

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both boundaries are free and rigid for different values of the parameter ε are displayed in Figs. (1) and (2) respectively for $N = 0.1$ and $N = 0.01$ when $P_r = P_r', P_m = 4$ and $Q = 5000$.

It is clear from these figures that the solute concentration has a destabilizing effect. Moreover as the permeability of porous medium, N, decreases the fluid becomes more stable in all cases of solute boundary conditions. Also it appears from these figures that the strength of the non-linear relation between the magnetic field and the magnetic induction, ε, makes the fluid more stable. In fact the strength of the non-linearity and the permeability of porous medium reinforce each other. It appears also that in case (1) the fluid becomes more stable than in cases (ii) and (iii). Similar relations between R_t and R_s are displayed in Figs. (3) and (4).

The relations between the magnetic parameter, Q, and the critical thermal Rayleigh number, R_t, for the overstability case for the three cases of solute boundary conditions when both boundaries are free and rigid are displayed in Figs. (5) and (6) respectively for $N = 0.1$ and $N = 0.01$ when $R_s = 3000$.

Similar relations between Q and R_s are displayed in Figs. (7) and (8) for $N = 0.1$ and $N = 0.01$ when $R_t = 3000$. It appears from these figures that as Q increases R_t and R_s increases which means that the presence of magnetic field makes the fluid more stable. Moreover as the permeability of porous medium, N, decreases the fluid becomes more stable in all cases of solute boundary conditions. Also it appears from these figures that the strength of the non-linear relation between the magnetic field and the magnetic induction, ε, makes the fluid more stable.

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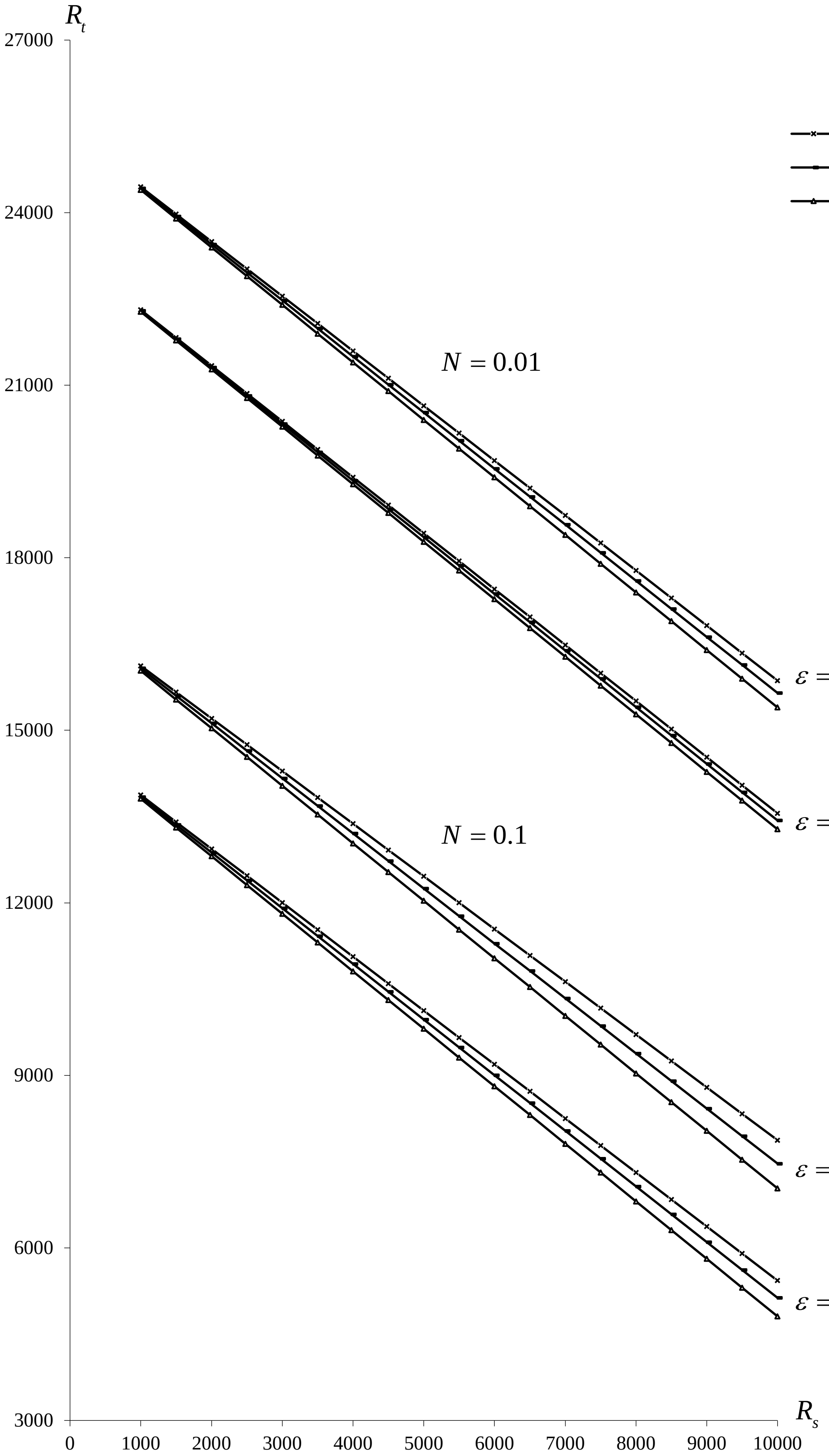

Fig 1: The relation between R_s and R_t for the overstability case and for the three cases of solute boundary conditions when both boundaries are free for Q = 5000.

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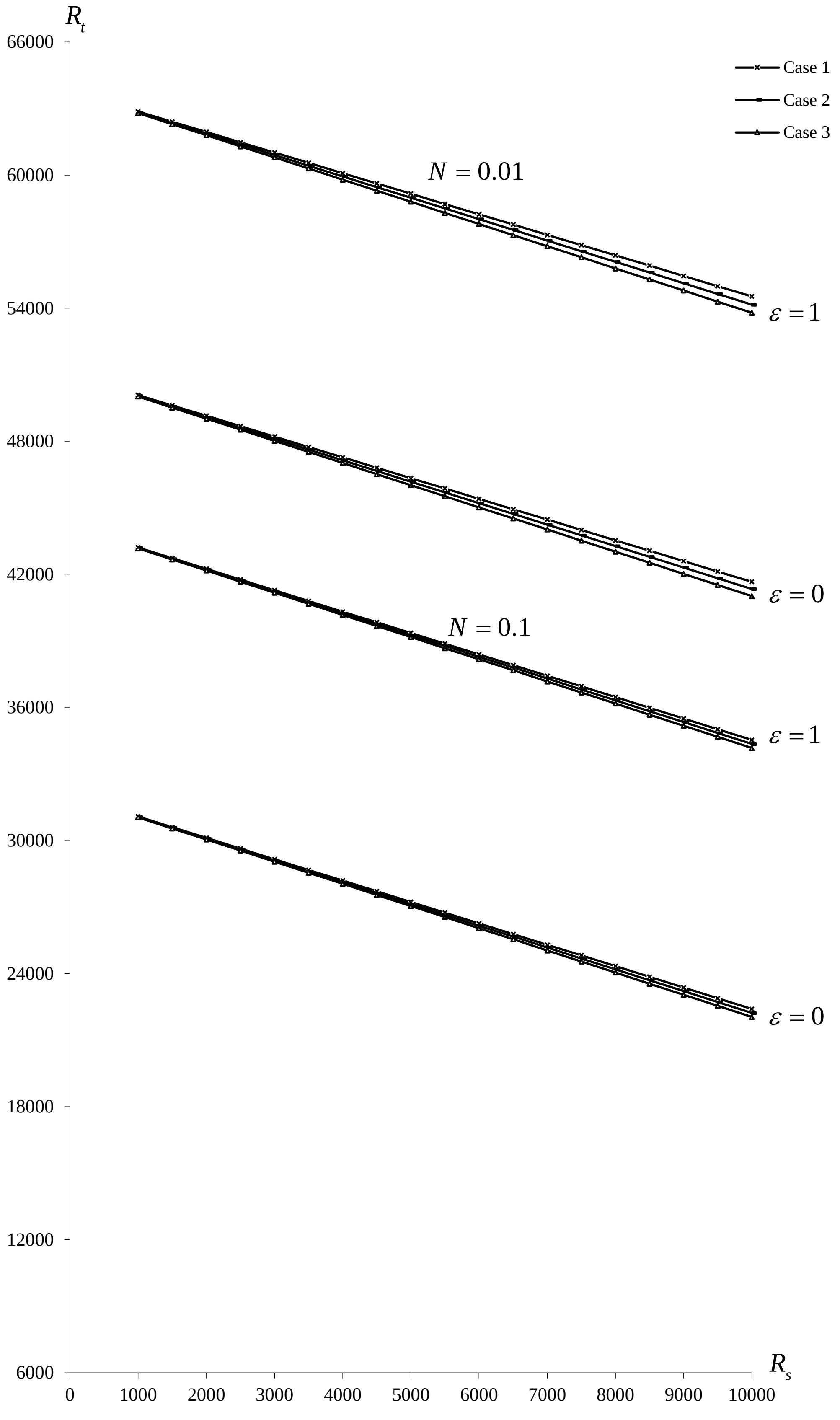

Fig 2: The relation between R_s and R_t for the overstability case and for the three cases of solute boundary conditions when both boundaries are rigid for Q = 5000.

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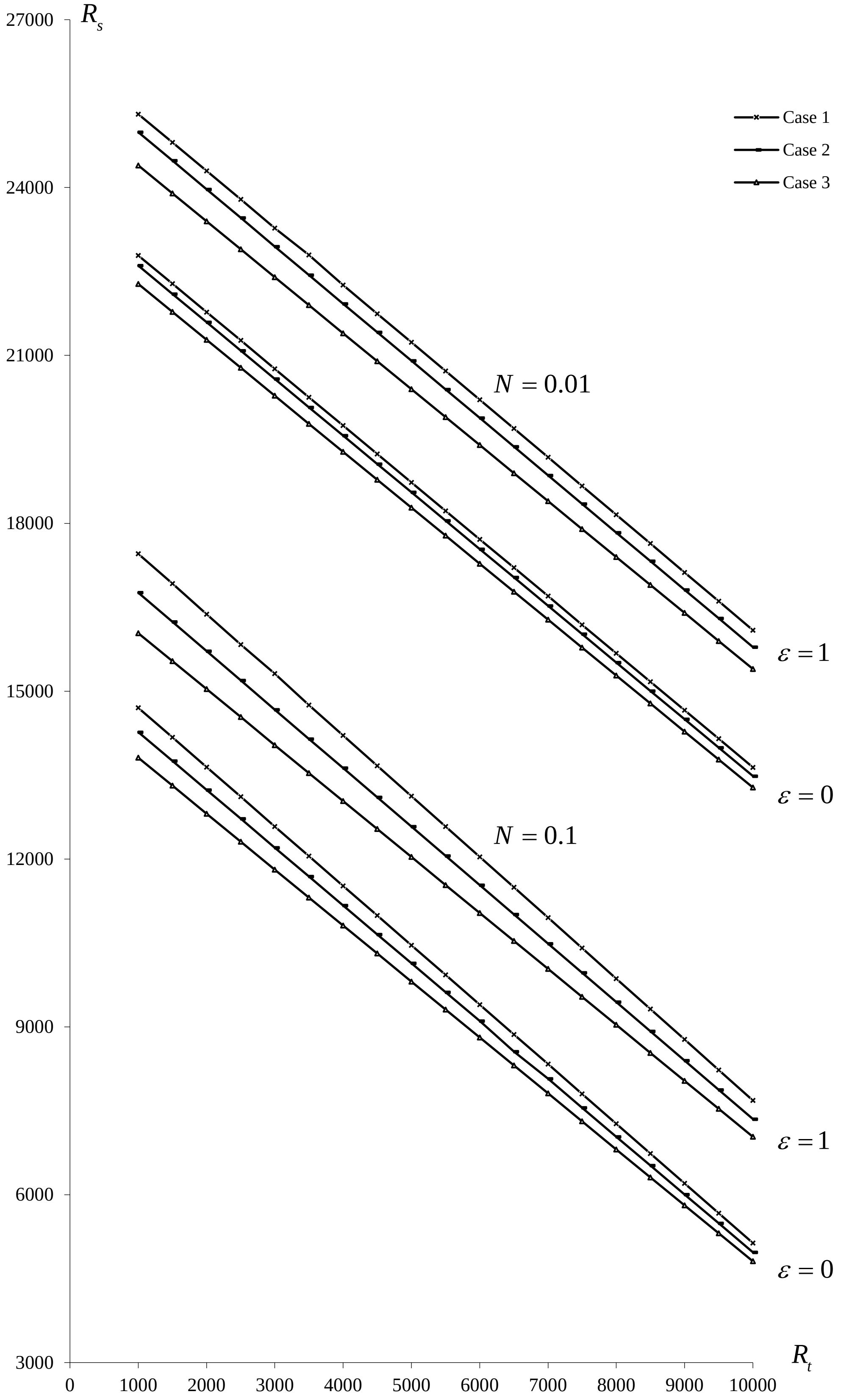

Fig 3: The relation between R_t and R_s for the overstability case and for the three cases of solute boundary conditions when both boundaries are free for Q = 5000.

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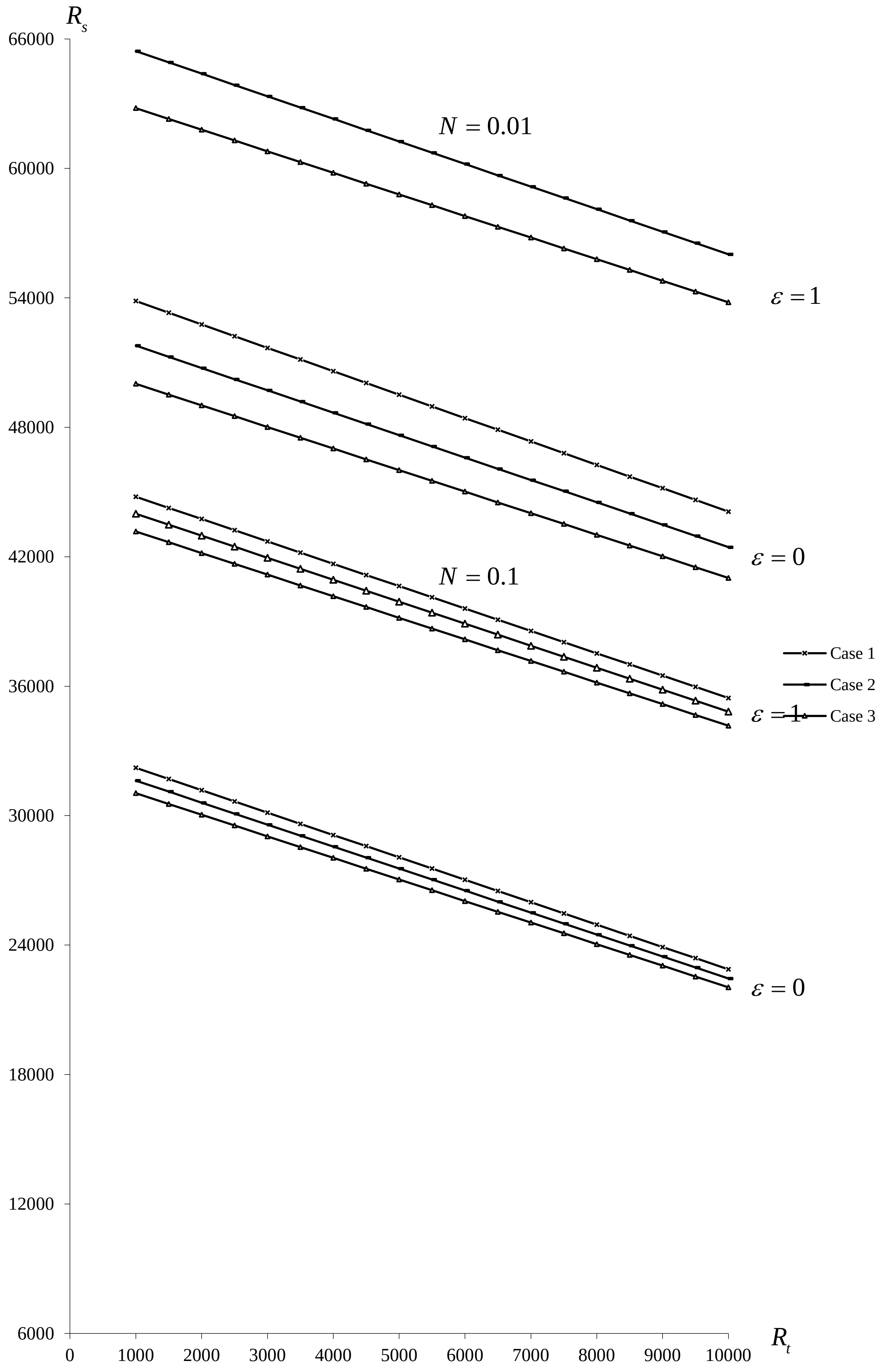

Fig 4: The relation between R_t and R_s for the overstability case and for the three cases of solute boundary conditions when both boundaries are rigid for Q = 5000.

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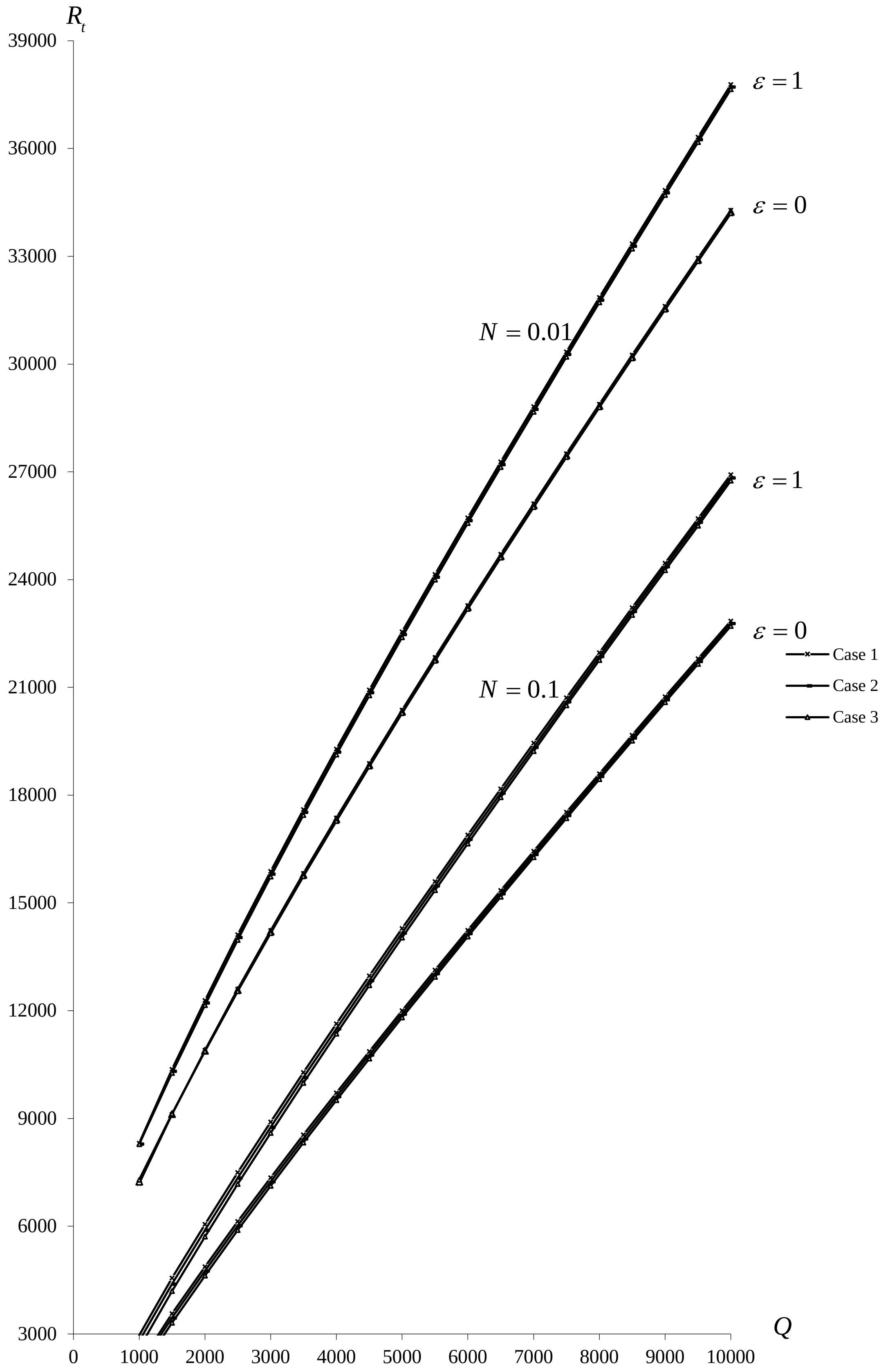

Fig 5: The relation between Q and R_t for the overstability case and for the three cases of solute boundary conditions when both boundaries are free for R_s= 3000.

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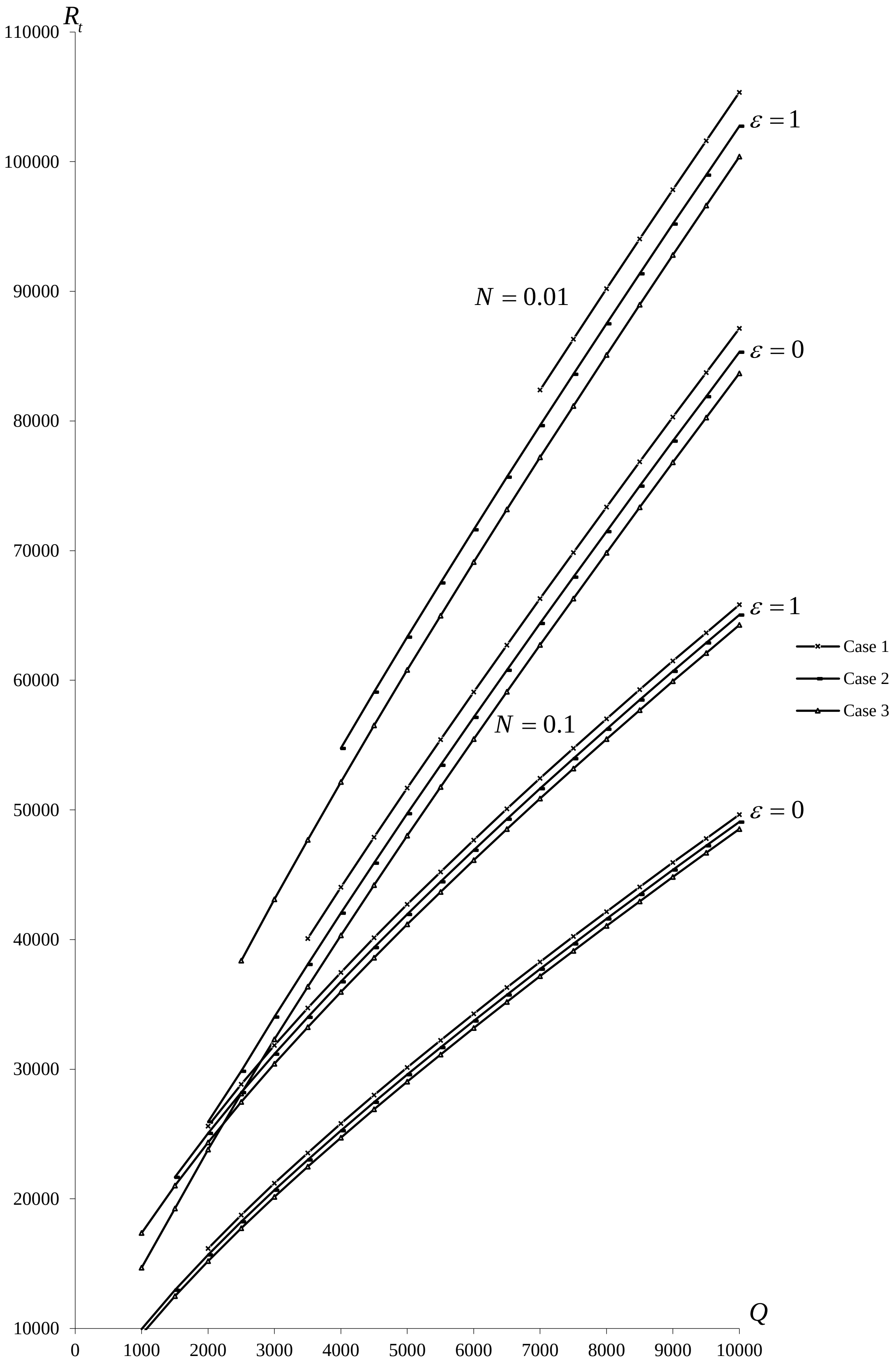

Fig 6: The relation between Q and R_t for the overstability case and for the three cases of solute boundary conditions when both boundaries are rigid for R_s= 3000.

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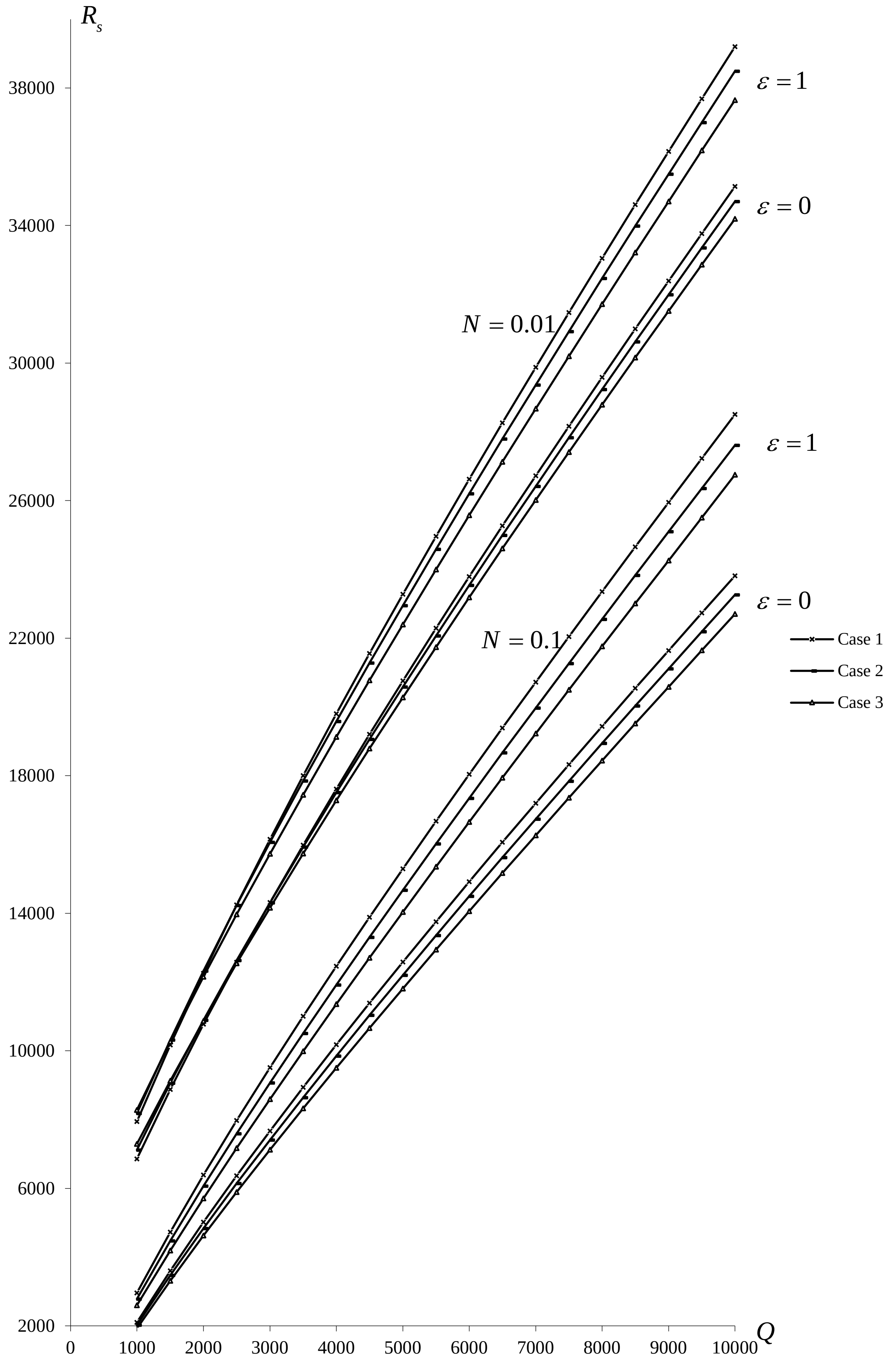

Fig 7: The relation between Q and R_s for the overstability case and for the three cases of solute boundary conditions when both boundaries are free for $R_t = 3000$.

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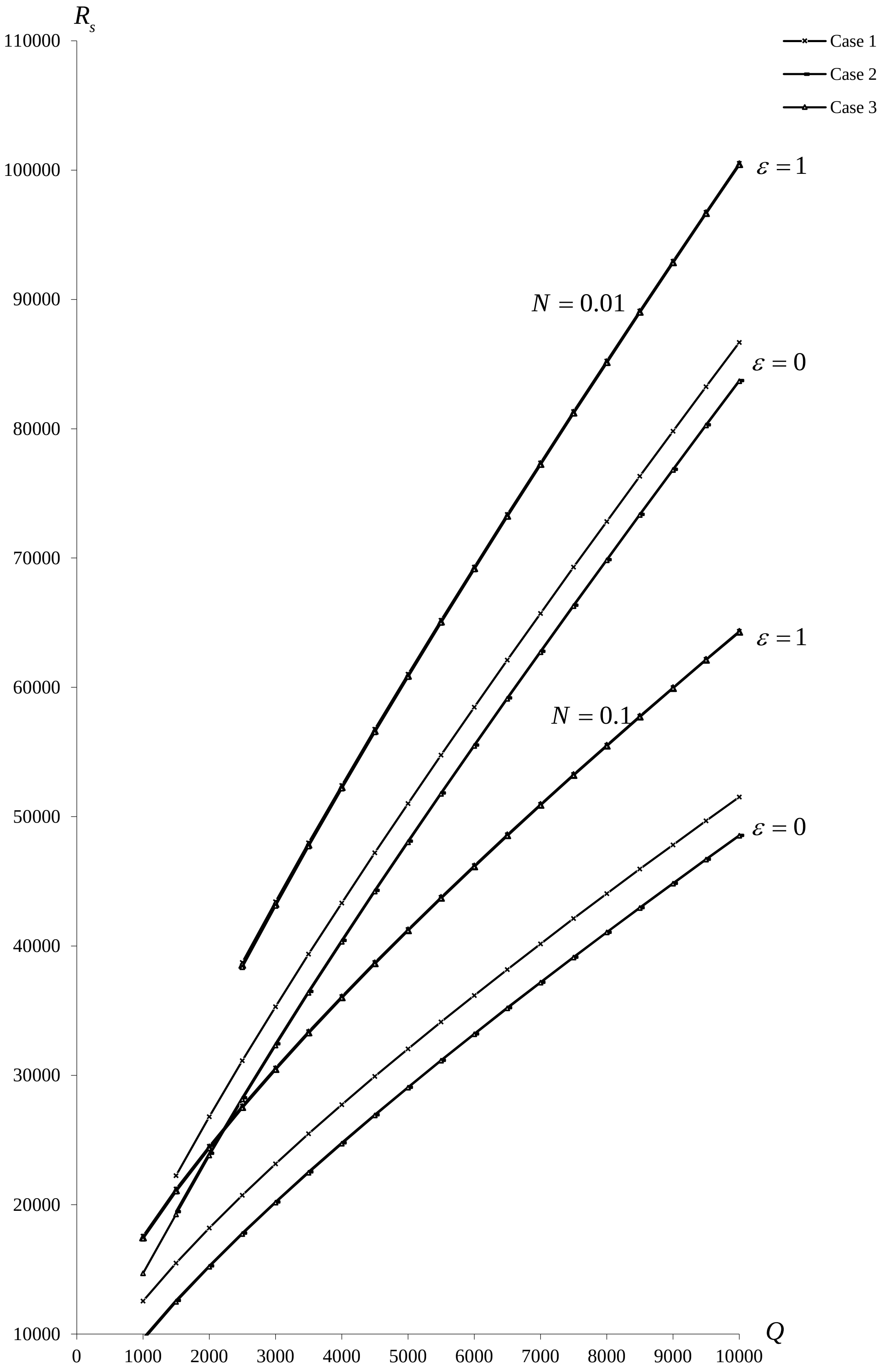

Fig 8: The relation between Q and R_s for the overstability case and for the three cases of solute boundary conditions when both boundaries are rigid for $R_t = 3000$.

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